

# Pattern Recognition

# Part 3: Beamforming

#### **Gerhard Schmidt**

Christian-Albrechts-Universität zu Kiel Faculty of Engineering Institute of Electrical and Information Engineering Digital Signal Processing and System Theory



Christian-Albrechts-Universität zu Kiel

### Contents

- Introduction
- □ Characteristic of multi-microphone systems
- Delay-and-sum structures
- □ Filter-and-sum structures
- Interference compensation
- Audio examples and results
- Outlook on postfilter structures







Christian-Albrechts-Universität zu Kiel

### Introduction – Part 1









### Literature

#### Beamforming

- E. Hänsler / G. Schmidt: Acoustic Echo and Noise Control Chapater 11 (Beamforming), Wiley, 2004
- □ H. L. Van Trees: *Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory*, Wiley, 2002
- W. Herbordt: Sound Capture for Human/Machine Interfaces: Practical Aspects of Microphone Array Signal Processing, Springer, 2005

#### Postfiltering

- K. U. Simmer, J. Bitzer, C. Marro: Post-Filtering Techniques, in M. Brandstein, D. Ward (editors), Microphone Arrays, Springer, 2001
- S. Gannot, I. Cohen: Adaptive Beamforming and Postfiltering, in J. Benesty, M. M. Sondhi, Y. Huang (editors), Springer Handbook of Speech Processing, Springer, 2007



CAU

Introduction – Part 2

#### **Basis structure:**



### **Difference equation:**

$$u(n) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} y_m(n-i) g_{m,i}(n)$$





### Introduction – Part 3

### Difference equation in vector notation:

$$u(n) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} y_m(n-i) g_{m,i}(n)$$
$$= \sum_{m=0}^{M-1} \boldsymbol{y}_m^{\mathrm{T}}(n) \boldsymbol{g}_m(n)$$

with

$$\boldsymbol{y}_{m}(n) = \begin{bmatrix} y_{m}(n), y_{m}(n-1), ..., y_{m}(n-N+1) \end{bmatrix}^{\mathrm{T}},$$
  
$$\boldsymbol{g}_{m}(n) = \begin{bmatrix} g_{m,0}(n), g_{m,1}(n), ..., g_{m,N-1}(n) \end{bmatrix}^{\mathrm{T}}.$$

#### For fixed (time-invariant) beamformers we get:

$$u(n) = \sum_{m=0}^{M-1} \boldsymbol{y}_m^{\mathrm{T}}(n) \, \boldsymbol{g}_m \quad \Longleftrightarrow \quad U(e^{j\Omega}) = \sum_{m=0}^{M-1} Y_m(e^{j\Omega}) \, G_m(e^{j\Omega})$$



### Introduction – Part 4

### Microphone positions and coordinate systems:





### Introduction – Part 5

### Directivity due to filtering and sensor characteristics:



Directivity can be achieved either by spatial filtering of the microphone signals according to

$$u(n) = \sum_{m=0}^{M-1} \boldsymbol{y}_m^{\mathrm{T}}(n) \, \boldsymbol{g}_m$$

or by the *sensors themselves* (e.g. due to cardioid characteristics).

If we use spatial filtering a *reference for the disturbing signal components* can be estimated. This can be exploited by means of, e.g. a *Wiener filter* and leads to an additional directivity gain.





## Quality Measures of Multi-Microphone Systems – Part 1

### Assumptions for computing a "spatial frequency response":

□ The sound propagation is modeled as *plane wave*:

 $S_m(e^{j\Omega}) = S(e^{j\Omega}) e^{-j\Omega\tau_m}.$ 

□ Each microphone has got a *receiving characteristic*, which can be described as

$$M_m(e^{j\Omega}, \mathbf{r}) = M_m(e^{j\Omega}, \varphi, \theta).$$

For microphones with omnidirectional characteristic the following equation holds,

$$M_{m,\text{omni}}(e^{j\Omega},\varphi,\theta) = 1.$$

Microphones with *cardioid characteristic* can be described as

$$M_{m,\text{card}}(e^{j\Omega},\varphi,\theta) = \frac{1}{2} \Big[ 1 + \cos(\varphi) \Big].$$





### Quality Measures of Multi-Microphone Systems – Part 2

### Spatial frequency response

□ With the above assumptions the *desired signal component of the output spectrum of a single microphone* can be written as

$$Y_m(e^{j\Omega}, \boldsymbol{r}) = S_m(e^{j\Omega}) M_m(e^{j\Omega}, \boldsymbol{r})$$
  
=  $S(e^{j\Omega}) M_m(e^{j\Omega}, \boldsymbol{r}) e^{-j\Omega\tau_m}.$ 

□ The *output spectrum of the beamformer* can consequently be written as

$$U(e^{j\Omega}, \boldsymbol{r}) = \sum_{m=0}^{M-1} Y_m(e^{j\Omega}, \boldsymbol{r}) G_m(e^{j\Omega})$$
  
=  $S(e^{j\Omega}) \sum_{m=0}^{M-1} M_m(e^{j\Omega}, \boldsymbol{r}) G_m(e^{j\Omega}) e^{-j\Omega\tau_m}.$ 

□ Finally the *spatial frequency response* is defined as follows,

$$G_{\rm BF}(e^{j\Omega},\boldsymbol{r}) = \frac{U(e^{j\Omega},\boldsymbol{r})}{S(e^{j\Omega})} = \sum_{m=0}^{M-1} M_m(e^{j\Omega},\boldsymbol{r}) \, G_m(e^{j\Omega}) \, e^{-j\Omega\tau_m}.$$





### Quality Measures of Multi-Microphone Systems – Part 3

#### **Examples of spatial frequency responses**



□ 4 microphones in a row in intervals of 3cm were used.





### Quality Measures of Multi-Microphone Systems – Part 4

#### Beampattern

□ The squared absolute of the spatial frequency response is called *beampattern*:

$$\Phi(\Omega, \boldsymbol{r}) = |G_{\mathrm{BF}}(e^{j\Omega}, \boldsymbol{r})|^2.$$

□ If all microphones have the same *beampattern*, the influences of the microphones and of the signal processing can be *separated*:

$$\begin{split} \Phi(\Omega, \boldsymbol{r}) &= \left| \sum_{m=0}^{M-1} M_m(e^{j\Omega}, \boldsymbol{r}) \, G_m(e^{j\Omega}) \, e^{-j\Omega\tau_m} \right|^2 \\ &= \left| M(e^{j\Omega}, \boldsymbol{r}) \right|^2 \left| \sum_{m=0}^{M-1} G_m(e^{j\Omega}) \, e^{-j\Omega\tau_m} \right|^2 \\ &= \Phi_{\mathrm{mic}}(\Omega, \boldsymbol{r}) \, \Phi_{\mathrm{sig}}(\Omega, \boldsymbol{r}). \end{split}$$





### Quality Measures of Multi-Microphone Systems – Part 5

#### Array gain:

□ If a characteristic number is needed, the so-called array gain can be used,

$$Q(\Omega, \boldsymbol{r}_{\mathrm{s}}) = rac{\Phi(\Omega, \boldsymbol{r}_{\mathrm{s}})}{rac{1}{A_{\mathrm{sphere}}} \int\limits_{\mathrm{surface}} \Phi(\Omega, \boldsymbol{r}) \, dA}.$$

 $\Box$  The vector  $r_{
m s}$  is pointing into the direction of the desired signal.

□ The logarithmic array gain

$$Q_{\log}(\Omega, \boldsymbol{r}_{\mathrm{s}}) = 10 \log_{10} \left\{ Q(\Omega, \boldsymbol{r}_{\mathrm{s}}) \right\}$$

is called *directivity index*.

Both quantities describe the gain compared to an onmidirectional sensor (e.g., a microphone with omnidirectional characteristic).



### Delay-and-Sum Structure – Part 1

### **Basic structure**



- The microphone signals are being delayed in such a way that all signals from a predefined preferred direction are synchronized after the delay compensation.
- In the next step, the signals are weighted and added in such a way that at the output, the signal power of the desired signal from the preferred direction is the same as at the input (but without reflections).
- Interferences which do not arrive from the preferred direction, will not be added in-phase and will therefore be attenuated.

## Delay-and-Sum Structure – Part 2

### Identify the necessary delays



 $\Box$  In the case of a linear array with constant microphone distance, the distance of the m<sup>th</sup> mid<sub>mic</sub>tone to the center of the array can be calculated as

$$d_m = \|\boldsymbol{r}_m\| = \left|m - \frac{M-1}{2}\right| d_{\mathrm{mic}}$$

Based on this distance, we can calculate the *time delay* of the plane wave to arrive at the m<sup>th</sup> microphone,

$$t_m = \frac{d_{\Delta,m}}{c} = \frac{d_m \, \sin(\varphi_s)}{c}.$$

□ Using the sample rate, the time delay can be expressed in frames,

 $\tau_m = t_m f_{\rm s}.$ 





### Delay-and-Sum Structure – Part 3

### **Optimal** solution

$$G_{\text{opt},m}(e^{j\Omega}) = e^{-j\Omega\tau_m} \text{ with } -\pi < \Omega \leq \pi \quad \bullet \quad O \quad g_{\text{opt},m,i} = \frac{\sin\left(\pi(i-\tau_m)\right)}{\pi(i-\tau_m)}$$

### Implementation in time domain (example)

□ The optimal impulse response is delayed to make it causal, and is then *"windowed*",

$$g_{m,i} = g_{\text{opt},m,i-N/2} f_i.$$

□ As window function, for example the Hann window can be chosen,

$$f_{i} = \begin{cases} K \left[ 1 + \sin\left(\frac{2\pi}{N}(i - \frac{N}{2})\right) \right], & \text{for } i \in \{0, ..., N - 1\}, \\ 0, & \text{else.} \end{cases}$$



## Delay-and-Sum Structure – Part 4

### Implementation in time domain (example)



□ Goal: Design a filter with group delay of 10.3 samples.

Constraint: 21 filter coefficients may be used.





### Delay-and-Sum Structure – Part 5



#### Implementation in the frequency domain



### Filter-and-Sum Structure – Part 1

### **Basic principle**



- □ In addition to the delay compensation, the array characteristic are to be improved using *filters*.
- As soon as the beamformer properties are better than the delay-and-sum approach, the beamformer is called *superdirective*.
- The introduced filters are designed to be optimal for the broadside direction as preferred direction.



### Filter-and-Sum Structure – Part 2

### Filter design

#### Difference equation:

$$u(n) = \sum_{m=0}^{M-1} \sum_{i=0}^{\tilde{N}-1} \widetilde{y}_m(n-i) \, \widetilde{g}_{m,i}$$
$$= \sum_{m=0}^{M-1} \widetilde{y}_m^{\mathrm{T}}(n) \, \widetilde{g}_m$$

Optimization criterion:

$$\mathrm{E}ig\{u^2(n)ig\} \longrightarrow \min$$
 with the constraint  $\sum_{m=0}^{M-1} \widetilde{oldsymbol{g}}_m = oldsymbol{c}$ 





### Filter-and-Sum Structure – Part 3

### Constraints of the filter design

M - 1 $\sum \widetilde{g}_m = c$ m=0 $\widetilde{oldsymbol{g}}_{0}\ \widetilde{oldsymbol{g}}_{1}$  $= \widetilde{g}_{0,0} \dots \widetilde{g}_{0,k-1} \qquad \widetilde{g}_{0,k} \qquad \widetilde{g}_{0,k+1} \dots \widetilde{g}_{0,\tilde{N}-1} \\ = \widetilde{g}_{1,0} \dots \widetilde{g}_{1,k-1} \qquad \widetilde{g}_{1,k} \qquad \widetilde{g}_{1,k+1} \dots \widetilde{g}_{1,\tilde{N}-1}$  $= \widetilde{g}_{M-1,0} \quad \dots \quad \widetilde{g}_{M-1,k-1} \quad \widetilde{g}_{M-1,k} \quad \widetilde{g}_{M-1,k+1} \quad \dots \quad \widetilde{g}_{M-1,\tilde{N}-1}$ • • •  $\widetilde{\boldsymbol{g}}_{M-1}$ 0 1 0 0 . . . 0 . . .  $\boldsymbol{c}$ =

# This means: Signals from the broadside direction can pass the filter network without any attenuation.

The "zero solution" is excluded by introducing the constraint!





### Filter-and-Sum Structure – Part 4

### Filter design

□ Introducing *overall signal vectors* and *overall filter vectors*:

 $\widetilde{\boldsymbol{g}} = [\widetilde{\boldsymbol{g}}_0^{\mathrm{T}}, \widetilde{\boldsymbol{g}}_1^{\mathrm{T}}, ..., \widetilde{\boldsymbol{g}}_{M-1}^{\mathrm{T}}]^{\mathrm{T}},$  $\widetilde{\boldsymbol{y}}(n) = [\widetilde{\boldsymbol{y}}_0^{\mathrm{T}}(n), \widetilde{\boldsymbol{y}}_1^{\mathrm{T}}(n), ..., \widetilde{\boldsymbol{y}}_{M-1}^{\mathrm{T}}(n)]^{\mathrm{T}}.$ 

□ Subsequently, the *beamformer output signal* can be written as follows:

$$u(n) = \widetilde{\boldsymbol{g}}^{\mathrm{T}} \widetilde{\boldsymbol{y}}(n) = \widetilde{\boldsymbol{y}}^{\mathrm{T}}(n) \widetilde{\boldsymbol{g}}.$$

□ The *mean output signal power* results in:

$$E\{u^{2}(n)\} = E\{\widetilde{\boldsymbol{g}}^{\mathrm{T}}\,\widetilde{\boldsymbol{y}}(n)\,\widetilde{\boldsymbol{y}}^{\mathrm{T}}(n)\,\widetilde{\boldsymbol{g}}\}$$
$$= \widetilde{\boldsymbol{g}}^{\mathrm{T}}\,E\{\widetilde{\boldsymbol{y}}(n)\,\widetilde{\boldsymbol{y}}^{\mathrm{T}}(n)\}\,\widetilde{\boldsymbol{g}}$$
$$= \widetilde{\boldsymbol{g}}^{\mathrm{T}}\,\boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}}\,\widetilde{\boldsymbol{g}}.$$





### Filter-and-Sum Structure – Part 5

### Filter design

□ The *constraint* can be rewritten as follows:

$$oldsymbol{c} = \sum_{m=0}^{M-1} \widetilde{oldsymbol{g}}_m \ = \left[oldsymbol{I}_{NxN}, oldsymbol{I}_{NxN}, ..., oldsymbol{I}_{NxN}
ight] \widetilde{oldsymbol{g}} \ = oldsymbol{C} \, \widetilde{oldsymbol{g}}.$$

□ Then, using a *Lagrange approach* the following function can be minimized:

$$F(\widetilde{\boldsymbol{g}}) = \frac{1}{2} \, \widetilde{\boldsymbol{g}}^{\mathrm{T}} \boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}} \, \widetilde{\boldsymbol{g}} + \boldsymbol{\lambda}^{\mathrm{T}} (\boldsymbol{C} \, \widetilde{\boldsymbol{g}} - \boldsymbol{c}).$$

 $\Box$  Calculating the *gradient* with respect to  $\widetilde{g}$  results in:

$$\nabla_{\widetilde{\boldsymbol{g}}} F(\widetilde{\boldsymbol{g}}) = \boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}} \, \widetilde{\boldsymbol{g}} + \boldsymbol{C}^{\mathrm{T}} \, \boldsymbol{\lambda}.$$





### Filter-and-Sum Structure – Part 6

### Filter design

□ Setting the gradient to zero results in:

$$\widetilde{\boldsymbol{g}}_{ ext{opt}} = - \boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}}^{-1} \, \boldsymbol{C}^{ ext{T}} \, \boldsymbol{\lambda}.$$

□ Inserting this result into the *constraint* we get:

$$oldsymbol{c} = oldsymbol{C} \Big( - oldsymbol{S}_{\widetilde{oldsymbol{y}}\widetilde{oldsymbol{y}}}^{-1} oldsymbol{C}^{ ext{T}} oldsymbol{\lambda} \Big).$$

□ Resolving this equation to the *Lagrange multiplication vector* results in:

$$oldsymbol{\lambda} = \left( - C \, oldsymbol{S}_{\widetilde{oldsymbol{y}}\widetilde{oldsymbol{y}}}^{-1} \, oldsymbol{C}^{\mathrm{T}} 
ight)^{-1} oldsymbol{c}$$

□ Finally, we get:

$$\widetilde{\boldsymbol{g}}_{ ext{opt}} = \boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}}^{-1} \, \boldsymbol{C}^{ ext{T}} \left( \boldsymbol{C} \, \boldsymbol{S}_{\widetilde{\boldsymbol{y}}\widetilde{\boldsymbol{y}}}^{-1} \, \boldsymbol{C}^{ ext{T}} 
ight)^{-1} \boldsymbol{c}.$$

The filter coefficients are defined by the auto correlation matrix of the interference sound field!



### Filter-and-Sum Structure – Part 7



- Goal: Design filters for a microphone array consisting of 4 microphones.
- □ The microphone distance is 4 cm.



### **Interference Cancellation**

#### **Basic principle**

- Up to now, we had to make assumptions about the properties of the sound field. If this is not possible, we should use an *adaptive error power minimization* instead.
- A direct application of adaptive algorithms would lead to the so-called "zero solution" (all filter coefficients are zero). So as before, we need to introduce a *constraint*.
- This constraint can either be taken care of when calculating the gradient (e.g., using the Frost approach), or implemented in the filter structure using a *desired signal blocking*. The latter is much more efficient.
- The desired signal blocking has the task to block the desired signal completely but to let pass all interferences. Using this output signal, a *minimization of the error power without constraints* can be applied.





### Interference Cancellation – Blocking the desired signal (part 1)

### Subtraction of delay-compensated microphone signals







### Interference Cancellation – Blocking the Desired Signal (Part 2)

#### Subtracting the delay-compensated microphone signals

#### Advantages:

- □ Very simple and *computationally efficient structure*.
- Besides just to subtract the signals, also the principles of filter design may be applied. Hereby, the width of the blocking can be controlled.

#### Drawbacks:

- In the case of errors in the delay compensation, or if different sensors are used, the desired signal may pass the blocking structure and may be *compensated* unintentionally.
- Echo components of the desired signal may pass the blocking structure, which may equally lead to a compensation of the desired signal.

#### **Conclusion:**

This blocking structure is usually used to classify the current situation (e.g., "desired signal active", "interference active", etc.). Based on this classification, further and more sophisticated approaches may be regulated.





### Interference Cancellation – Blocking the Desired Signal (Part 3)

#### Adaptive subtraction of delay-compensated microphone signals







### Interference Cancellation – Blocking the Desired Signal (Part 4)

#### Adaptive subtraction of delay-compensated microphone signals

#### Advantages:

- *Errors* in the delay compensation *may be compensated* (provided that the situation was classified correctly).
- *Echo components* can be (partly) removed.
- □ The structure can be used to *localize* the desired speaker (topic for a talk...)

#### Drawbacks:

- □ In the adaption, a *constraint* has to be fulfilled (e.g., the sum of the norms of the filters has to be constant).
- □ A *robust control* of the filter adaption is necessary.





### Interference Cancellation – Blocking the Desired Signal (Part 5)

### Adaptive subtraction of delay-compensated microphone signals and beamformer output







### Interference Cancellation – Blocking the Desired Signal (Part 6)

#### Adaptive subtraction of delay-compensated microphone signals and beamformer output

#### Advantages:

- *Echo components* can be (party) removed.
- □ The reference signal of the desired speaker (beamformer output) has a *better signal-to-noise ratio* than using the adaptive microphone signal filtering.
- □ Only one signal has to be kept in memory (*less memory requirements* than the structure before).

#### Drawbacks:

- □ To approximate the inverse room transfer function, usually *more parameters* are necessary (compared to direct approximation).
- □ A *robust control* of the filter adaption is necessary.





### Interference Cancellation – Blocking the Desired Signal (Part 7)

*Differences between the blocking structures:* 





$$H_2(e^{j\Omega}) = \left[H_1(e^{j\Omega}) + H_2(e^{j\Omega})\right] B_2(e^{j\Omega})$$

$$\implies B_2(e^{j\Omega}) = \frac{H_2(e^{j\Omega})}{H_1(e^{j\Omega}) + H_2(e^{j\Omega})}$$

The approximation of inverse impulse responses is necessary (zeros-only model)!



 $H_1(e^{j\Omega}) B_1(e^{j\Omega}) = H_2(e^{j\Omega}) B_2(e^{j\Omega})$ 

 $\implies B_1(e^{j\Omega}) = H_2(e^{j\Omega}) C(e^{j\Omega})$ 

 $B_2(e^{j\Omega}) = H_1(e^{j\Omega}) C(e^{j\Omega})$ 



### Interference Cancellation – Blocking the Desired Signal (Part 8)

#### Double-adaptive subtraction of microphone signals and beamformer output





### Interference Cancellation – Blocking the Desired Signal (Part 9)

#### Double-adaptive subtraction of microphone signals and beamformer output

#### Advantages:

- □ *Echo components* can be (partly) removed.
- The reference signal of the desired speaker (*beamformer output*) has a *better signal-to-noise ratio* than using the adaptive microphone signal filtering.
- □ The approximation of inverted transfer functions is not necessary.

#### Drawbacks:

- □ A *robust control* of the filter adaption is necessary.
- □ Again, we need to *normalize* (at least one) filter norm.





### Audio Examples and Results – Part 1

- 4-channel microphone array
- Directional noise source (loudspeaker of the vehicle)
- Noise suppression > 15 dB by adaptive filtering of the microphone signals





Single microphone

Fixed beamformer

Adaptive beamformer



### Audio Examples and Results – Part 2

### Recognition rates of a dialog system



- □ Noise and speech have been added with different weights
- □ Speech model with 40 command words for radio and telephone applications
- □ 16 speakers (9 male, 7 female)





### Postfiltering – Part 1

### Previous structure (excerpt in subband domain)





# CAU

Christian-Albrechts-Universität zu Kiel

### Postfiltering – Part 2

### Extended structure (excerpt in subband domain)





## C A U Christian-Albrechts-Universität zu Kiel

## Beamforming

### Postfiltering – Part 3



#### **Boundary conditions:**

Two (ideal) omnidirectional microphones

□ Microphone distance 10 cm



### Postfiltering – Part 4

### **Boundary conditions**

- Microphone array consisting of 4 microphones.
- □ While the recording, the direction indicator is active

#### Results

- The sound of the direction indicator can be removed during speech pauses.
- During speech activity, the indicator sound can be removed only partly.





## Postfiltering – Part 5

### **Boundary conditions**

- □ Microphone array consisting of 4 microphones.
- The passenger says the name of a city, where after the driver repeats the name of the city.





### Summery and Outlook

#### Summary:

#### Introduction

- Quality measures for multi-microphone systems
- Delay-and-sum schemes
- □ Filter-and-sum schemes
- □ Interference cancellation
- □ Audio examples and results
- Post-filter schemes

### Next part:

Feature extraction





Christian-Albrechts-Universität zu Kiel