ADSP - Exercise 1: Peer Oral-Examination Cards

Instructions

Work in pairs.

- Round 1, 3 min: Student A is the examiner and uses card A.
- Round 2, 3 min: Change roles; student B is the examiner and uses card B.
- Feedback, 2 min: Use the checklist at the end.

The examiner may use the notes printed on the card. Do not show the expected-answer notes to the examinee during the answer.

Card A - Analog input, sampling, and aliasing

Questions to ask

Main question

Sketch and explain the signal path from the continuous-time input up to the output of the digital filter.

Follow-up questions

1. Where can aliasing arise, and why is it irreversible?
2. Why can a digital lowpass not replace the analog anti-aliasing filter?
3. Why does a realizable anti-aliasing filter require a transition band?
4. How are the analog angular frequency ω and the normalized digital frequency Ω related?

Choose two follow-up questions, depending on the first answer.

Notes for the examiner

A complete signal path contains

$$v_\mathrm{a}(t) \rightarrow H_\mathrm{AA}(j\omega) \rightarrow \text{sample and hold / ideal sampling} \rightarrow \text{AD converter} \rightarrow v(n) \rightarrow H(e^{j\Omega}) \rightarrow y(n).$$

Quantization in the AD converter is omitted in this exercise.

Expected key ideas:

- Sampling creates replicas centered at integer multiples of $\omega_\mathrm{s} = 2\pi/T$.
- Aliasing means that these replicas overlap. Contributions from different original frequencies then occur at the same discrete-time frequency.
- A digital filter operates after the overlap has occurred and cannot determine the original source of an aliased component.
- The anti-aliasing filter must limit the analog bandwidth before sampling to $|\omega| < \omega_\mathrm{s}/2$.
- An ideal brick-wall filter is not realizable. A real filter needs frequency space between its passband and stopband edges; this motivates a guard band below $\omega_\mathrm{s}/2$.
- Frequency mapping:

$$\Omega = \omega T = 2\pi \frac{\omega}{\omega_\mathrm{s}} = 2\pi \frac{f}{f_\mathrm{s}}.$$

Useful prompt if the examinee is stuck:

> Imagine that a 6 kHz component is sampled at 8 kHz and appears at 2 kHz. What information would a digital filter need to decide whether that 2 kHz component was originally at 2 or 6 kHz?

Card B - D/A conversion, reconstruction, and equivalent system

Questions to ask

Main question

Sketch and explain the signal path from the output of the digital filter to the reconstructed continuous-time output. Then explain how the complete chain can be represented by an equivalent analog frequency response.

Follow-up questions

1. What does the sample-and-hold circuit do to the magnitude and phase response?
2. What is the task of the reconstruction filter?
3. Which sample-and-hold effect is, in principle, compensable, and how?
4. Under which conditions does the equivalent analog frequency response exist?

Choose two follow-up questions, depending on the first answer.

Notes for the examiner

A complete output-side path contains

$$y(n) \to \text{DA converter} \to h_{\text{sh}}(t) \to y_{\text{sh}}(t) \to h_{\text{r}}(t) \to \hat{y}_{\text{a}}(t).$$

The sample-and-hold response (lecture notes, slide 21) is

$$H_{\text{sh}}(j\omega) = T\,\text{si}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2}, \qquad \text{si}(x) = \frac{\sin x}{x}.$$

Expected key ideas:

- The si factor produces an amplitude droop toward high frequencies (about $-3.9\,\text{dB}$ at $\omega_{\text{s}}/2$).
- The exponential is a linear phase term and represents a delay of $T/2$.
- The reconstruction filter suppresses spectral images and smooths the staircase-like signal $y_{\text{sh}}(t)$.
- Conversion of the discrete sequence to a continuous-time impulse train produces the spectral images; the sample-and-hold circuit weights them with its si-shaped response.
- The droop is deterministic and can in principle be equalized where the si response is nonzero, e.g. by pre-biasing the reconstruction filter (lecture, slide 22) or by a digital inverse-si pre-equalizer.
- An acceptable complete expression is

$$H_{\text{eff}}(j\omega) = H_{\text{AA}}(j\omega)\, H\left(e^{j\omega T}\right) \frac{1}{T} H_{\text{sh}}(j\omega)\, H_{\text{r}}(j\omega), \qquad |\omega| < \frac{\omega_{\text{s}}}{2}.$$

- In the convention used in this exercise, ideal impulse-train sampling contributes $1/T$, the ideal DA converter has unit gain, and the sampling factor cancels the factor T in H_{sh}. This cancellation should be explained explicitly.
- Conditions: no aliasing at the input ($V_{\text{a}} H_{\text{AA}} = 0$ for $|\omega| \geq \omega_{\text{s}}/2$) and no remaining images at the output ($H_{\text{r}} = 0$ for $|\omega| \geq \omega_{\text{s}}/2$). With real filters, H_{eff} is an approximation.

Useful prompt if the examinee is stuck:

> Which block acts before sampling, which block acts on the discrete-time sequence, and which two blocks affect the D/A output?

Peer feedback checklist

For each item, mark **secure**, **partly secure**, or **not yet clear**.

Criterion	Secure	Partly secure	Not yet clear
The block diagram is complete and correctly ordered.	☐	☐	☐
The frequency mapping $\Omega = \omega T$ is explained.	☐	☐	☐
The roles of anti-aliasing, digital, and reconstruction filters are distinguished.	☐	☐	☐
Aliasing is identified as irreversible spectral overlap.	☐	☐	☐
Sample-and-hold droop and linear phase are interpreted.	☐	☐	☐
The equivalent response is complete, and its conditions of validity are named.	☐	☐	☐
The explanation is clear and uses appropriate technical language.	☐	☐	☐

Give one short positive observation and one suggestion for improvement.